



## Experimental investigations of Poiseuille number laminar flow of water and air in minichannels

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### ABSTRACT

This paper presents the results of experiential investigations of pressure drop in minichannels, with use of water and air as the working fluids. The test section was made from stainless steel pipes with internal diameters of 0.55, 0.64 and 1.10 mm, respectively. A pressure drop was presented per a length unit as the function of Reynolds number. A comparison of the experimental friction factor with the results obtained from theoretical equations of Hagen–Poiseuille was presented. The experiments were conducted in range of Reynolds number  $Re = 30$  up to transition to the turbulent flow.

Contradictory reports concerning a sooner transition to the turbulent flow, or the friction factor values which diverge from those occurring in conventional channels, were not confirmed here.

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### 1. Introduction

Together with the technological advancement, heat exchangers of a size much smaller than that of exchangers generally known as conventional are becoming more and more common. Sample areas of the application of this type of exchangers are as follows: car industry – air conditioning systems; electronic industry – cooling of elements generating heat, fuel cells; housing industry: air conditioners, heat pumps. The application of heat exchangers of this type is becoming the more common the more their sizes are reduced, and the manufacturing technology more available.

Heat exchangers in which channels of this type are used have their advantages. These include the following: the possibility of work with higher working medium pressures, a much greater contact zone of the medium with the channel wall in relation to the liquid volume unit, a substantially smaller mass of the refrigerant applied, experimentally confirmed higher heat transfer coefficients, a lower wear of materials, or a smaller weight of the whole system. In the case of a depressurization of the system, the mass of the medium which will get to the environment is very small, which is advantageous both to a decrease of gases which cause the greenhouse effect and a degradation of the ozone layer. A drawback of compact heat exchangers is an increase of flow resistances together with a decrease of the hydraulic diameter of the channel as a result of the relative roughness increase. Another disadvantage

is, with favorable conditions, a fluctuation of the flow cause by an abrupt increase of bubbles.

In spite of the existence of numerous experimental and theoretical examinations, a certain number of the main hydrodynamic aspects have not received sufficient research. There are contradictory data concerning flow in this type of channels, such as the criterion of transition from the laminar to turbulent flow, or compatibility of the friction factor in laminar and turbulent flow with Hagen–Poiseuille law and Blasius equation respectively, valid for channels of classical sizes. This leads to difficulty of understanding the idea of the phenomena and constitutes the basis for very disputable discoveries. Discrepancies between the data have been interpreted as a disclosure of unknown effects of flow in small channels. However, the reason may be the fact that the conditions of the experiments were not identical with the conditions which were used for the formulation of theoretical models.

### 2. Studies of single-phase flow in minichannels

Pehlivan [1] made experimental investigations of the pressure drop of two-phase flow of water–air in channels with a circular section with an internal diameter of 3, 1 and 0.8 mm. In the part concerning pressure drop of single-phase flow, he confirmed both for water and air a compliance of Poiseuille conventional theory for laminar flow and Blasius theory for turbulent flow with the experimental results for minichannels.

Celata [2] examined the effect of a wall surface on the behavior of a liquid flowing inside channels with a circular section and internal diameters from 0.326 to 0.070 mm. The friction factor obtained

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### Nomenclature

$C^*$	normalized friction coefficient
$d$	internal diameter (m)
$L$	length (m)
$\dot{m}$	mass flow rate (kg/s)
$\Delta p$	pressure drop (Pa)
$Po$	Poiseuille number
$Re$	Reynolds number
$w$	mean velocity (m/s)
$x$	distance from inlet to pipe (m)
$x^+$	dimensionless length

### Greek symbols

$\lambda$	friction factor
$\mu$	dynamic viscosity (Ns/m <sup>2</sup> )
$\nu$	kinematic viscosity (m <sup>2</sup> /s)
$\rho$	density (kg/m <sup>3</sup> )

could in every case be correctly predicted by the dependencies valid for conventional channels.

Similar conclusions are presented in papers [3–6].

Hwang [7] made an investigation of pressure drop in circular minichannels made from stainless steel. In a review of the literature, he provides a tabular list of studies from which it is evident that the friction factor is larger than it results from theory (five studies); compliant with the theory (four studies) or smaller (four studies). The results of his experiments conducted on pipes with internal diameters of 0.244, 0.430 and 0.792 mm were compliant with the forecasts of the classical theories, while the transition flow started with  $Re$  slightly smaller than 2000.

Hetsroni [8] in his survey article presents a list of results of experimental investigations of the single-phase flow through smooth and rough channels with various shapes of cross sections. Hydraulic diameters were from 4.010 to 0.003 mm. The boundaries of a transition from laminar to turbulent flow occurred from  $Re = 300$  to  $Re = 3800$ , while the value of Poiseuille's number exceeded the theoretical values by even as much as 37%.

Wang [9] examined the friction coefficient during the flow of water and lubricating oil through channels with a circular and rectangular cross sections with hydraulic diameters  $d_h = 2.01$ – $0.198$  mm. The experimental values obtained of the friction factor for oil in rectangular channels were 10–30% lower, while for water slightly lower from those obtained in the theoretical way.

Agostini [10] made research into the heat transfer coefficient and the pressure drop of liquid R134a flowing in rectangular channels with hydraulic diameters of 1.17 and 0.77 mm. The results obtained of the investigation into the flow pressure drop were substantially higher (up to 50%) than those obtained from Shah-London's equation for laminar flow and from Blasius equation for turbulent flow, especially for the channel with greater dimensions. In paper [11], Agostini presents the result for flow R134a through eleven parallel channels with a rectangular section ( $d_h = 2.01$  mm). As the laminar single-phase flow occurred, a good agreement was obtained with Shah and London's correlation, and as turbulent flow occurred, with Colebrook's equation.

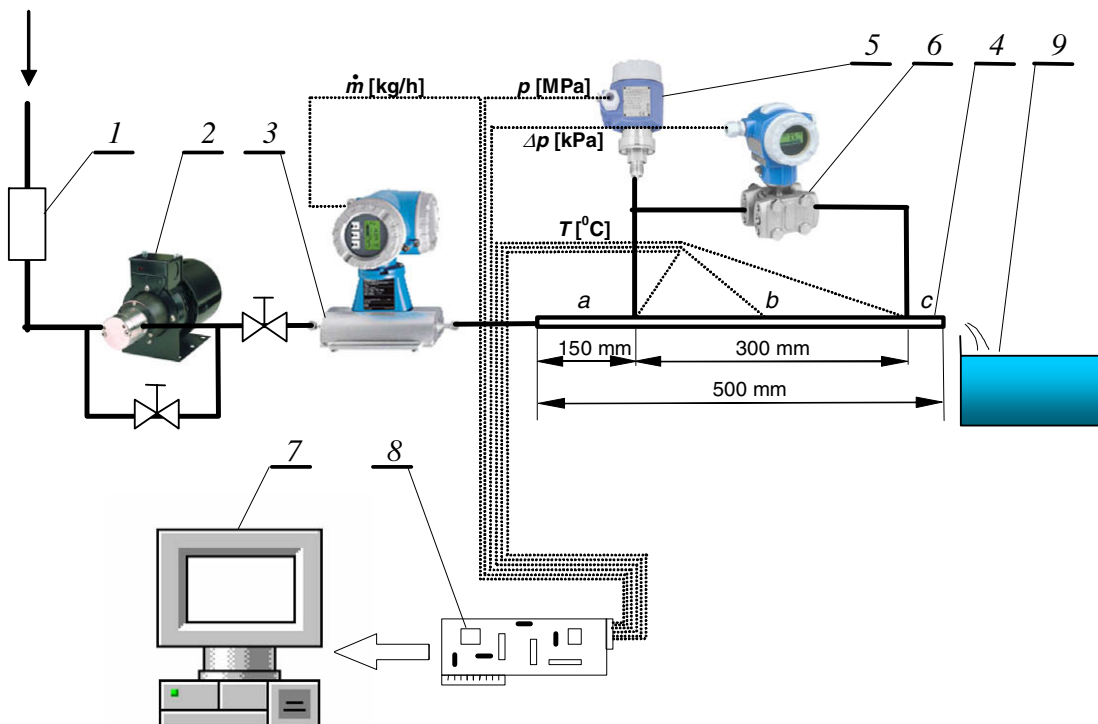


Fig. 1. Experimental set-up: 1 – filter, 2 – pump with valves (compressor if air flow) and adjustment valve, 3 – flow meter, 4 – test section, 5 – pressure transducer, 6 – difference pressure transducer, 7 – computer, 8 – measurement card, 9 – tank; (a–c) zones of the test section.

The paper by Shuai [12] includes results of the measurements of the friction coefficient for single-phase water flowing through channels with a rectangular section, corresponding to hydraulic diameters of 2.67 and 0.8 mm. The values obtained for the larger channel matched the values obtained from Hagen–Poiseuille equation and Blasius equation. The results for the channel with the smaller diameter exceed the theoretical values by even as much as 100%.

### 3. Test section

The experimental set-up (described for water flow) consists of a filter of particulates (1), a magnetic mini gear pump with a by-pass (2) and adjustment valve, a Coriolis mass flow-meter (3), a test section (4–6) connected to a data acquisition system (7, 8) and an outlet tank (9). Fig. 1 shows the experimental set-up.

The water from the tap flew through the filter of particulates. Its purpose was to prevent both the flow meter and the test section from being damaged. The water without particulates flew by the

by-pass or, if a higher pressure was required on the input to the test section, through a magnetic pump (D Series Magnetically Coupled Gear Pump manufactured by Tuthill Corporation) to the flow-meter. Coriolis mass flow-meter (Promass 80A manufactured by Endress–Hauser) was used with the measuring range of 0–20 kg/h. The measuring accuracy of this device is  $\pm 0.15\%$  of the measured value. With the flow intensity of 20 kg/h, this gives a measuring error of  $\pm 0.03$  kg/h. Further, the water with the already known efficiency flew to the test section.

The test section was composed of exchangeable stainless steel pipes. The whole pipe length was 500 mm, while its internal diameters (according to the manufacturer) were 0.55, 0.64 and 1.10 mm, respectively. In accordance with the classification proposed by Kandliakar [13–16], channels with these diameters can be considered to be minichannels. On the pipes, in the distances of 150 mm from the front and 50 mm from the end of the minichannel, small cuts were made with a milling cutter. The cuts were made in compliance with the remarks concerning execution of experiments in channels with diameters small than those

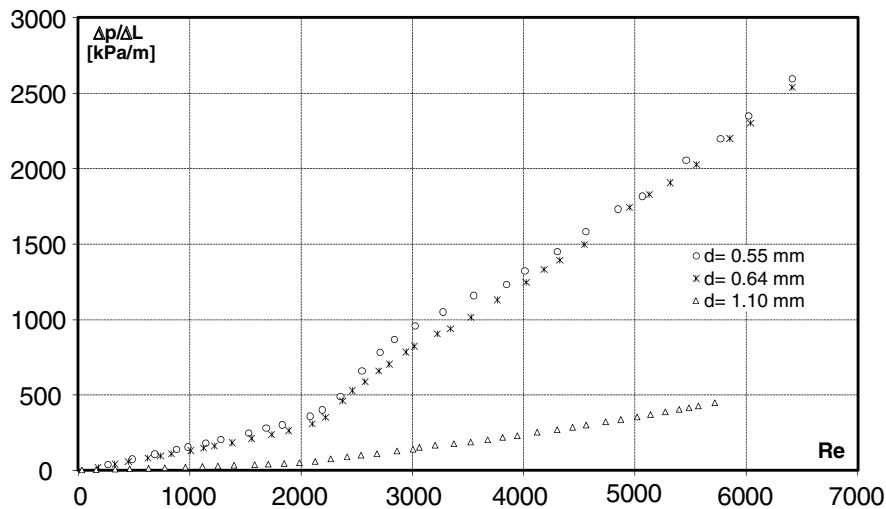


Fig. 2. Experimental flow resistance of water vs.  $Re$  number in minichannels with internal diameters: 0.55, 0.64 and 1.10 mm.

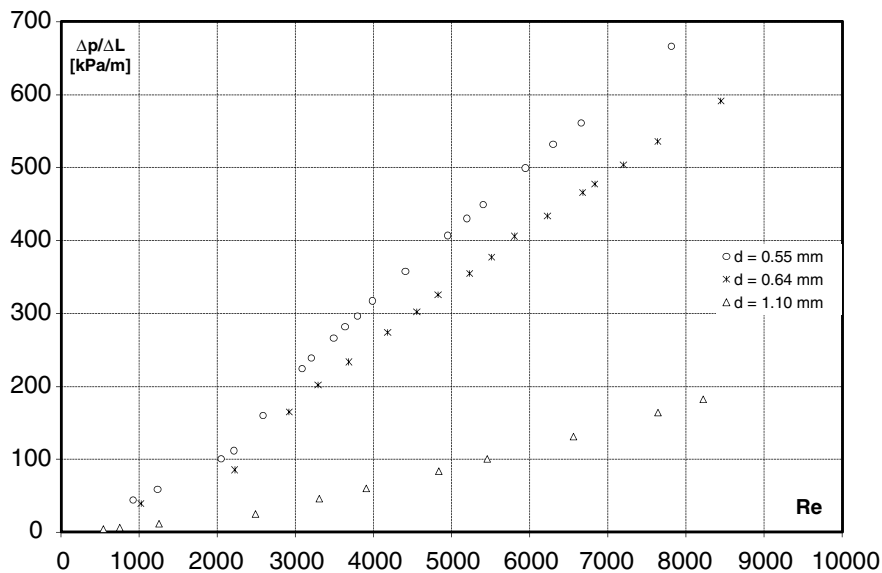


Fig. 3. Experimental flow resistance of air vs.  $Re$  number in minichannels with internal diameters: 0.55, 0.64 and 1.10 mm.

conventional ones ([15]), and therefore in such a manner so as to make it possible to receive the pressure impulse while not interfering the flow inside the minichannel. In this way, the whole 500 mm minichannel was divided into three sections. The first 150 mm section “a” stabilized the flow, the second – insulated – 300 mm section “b” constituted the measurement section, and the third 50 mm section was the outlet section – “c”. Water, while leaving the minichannel, flew into an open tank, and so its pressure corresponded with the present atmospheric pressure.

In accordance with the equation ([17])

$$x^+ = \frac{x}{d_h \cdot Re} \quad (1)$$

where  $x^+$  is a dimensionless hydrodynamic length,  $x$  is a length measured from the channel inlet; the flow can be considered to be fully

developed when dimensionless length  $x^+ = 0.05$  ( $x^+ = 0.055$  – according to Celata [2]). Accepting  $Re = 2000$  and the hydraulic diameters applied during the examinations, i.e.  $d_h = 0.55, 0.64$  and  $1.10$  mm the required entrance length is 55, 64 and 110 mm, respectively. A section of 150 mm used on the experimental set-up is sufficient to state that in the measuring section flow is fully developed.

Through the T-junction, which was fixed at the first hole, it is possible to measure the pressure at the input to the measuring section and one of impulses necessary for the measurement of the pressure drop. The second impulse, through the T-junction, is received from the next hole. For the pressure measurement, a piezoelectric sensor with a transducer (Cerabar M PMP41 manufactured by Endress–Hauser) was applied. This sensor has a measuring range of 0–1 MPa, and its accuracy does not exceed 0.2% of the

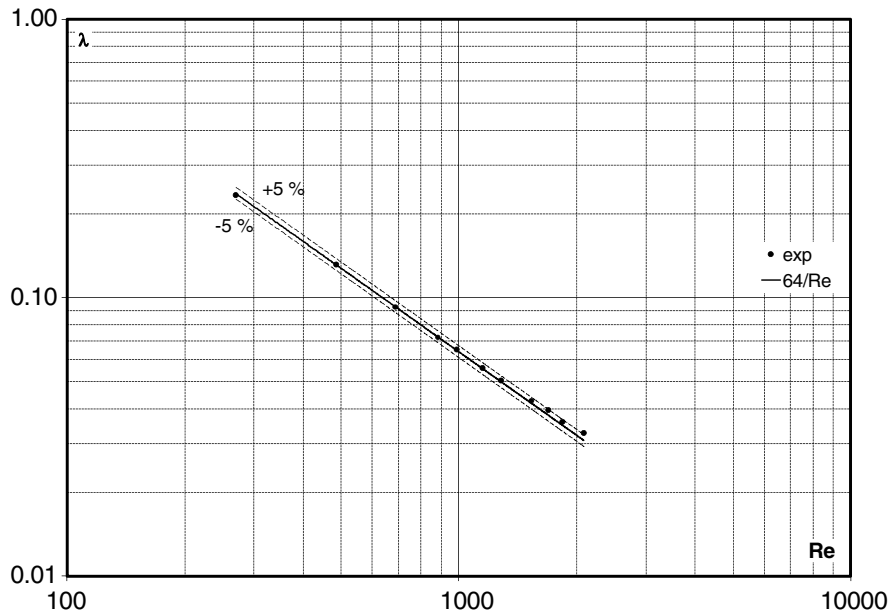


Fig. 4. Experimental frictional factor  $\lambda$  vs.  $Re$  number for water flow in minichannel with internal diameter  $d = 0.55$  mm.

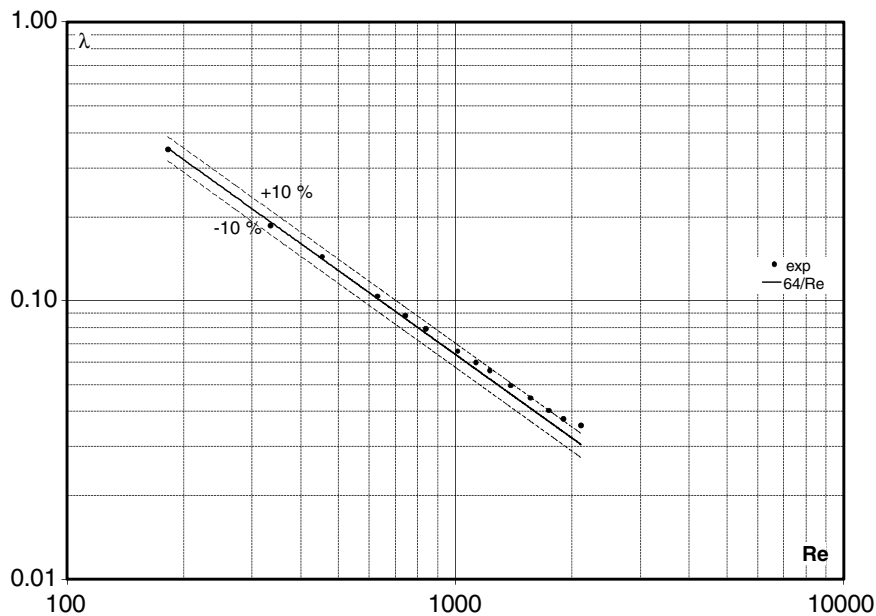


Fig. 5. Experimental frictional factor  $\lambda$  vs.  $Re$  number for water flow in minichannel with internal diameter  $d = 0.64$  mm.

measurement range. This gives a pressure measuring error of  $\pm 2$  kPa.

A pressure drop on the pipe length was measured with a piezo-resistive pressure difference sensor with a transducer (Deltabar S PMD75 manufactured by Endress–Hauser). The factory measuring range of the device is 0–500 kPa. The measuring accuracy for this device is 0.075% of the measuring range set. In the measuring range of 0–500, the measuring error is  $\pm 0.375$  kPa.

The flow-meter, the pressure sensor and the difference pressure sensor were individually calibrated by the manufacturer.

The temperature of the liquid flowing inside a minichannel on the length of the measuring section was measured with three K type thermocouples with a thickness of 0.2 mm. The thermocouples were individually calibrated in the range of 10–30 °C with

an accuracy of  $\pm 0.1$  °C, and were soldered on the length of 300 mm of the test section; right after the beginning, in the middle of the length, and right before the end. The whole was separated from the environment with 10 mm thick silicone insulation. The flow process was considered to be adiabatic. The thermal and physical properties of the fluid were read with the use of Bonca [18].

Data from the mass flow-meter, the pressure on the input of the measuring section, the pressure drop on the length of the test section of the minichannel and the temperature, were all registered by a data acquisition system made up of 16 bit, 1 MHz DaqBoard 3005 measuring card working with a PC.

The measurement procedure comprised the following stages. The mass flow rate was set with the aid of a valve located before

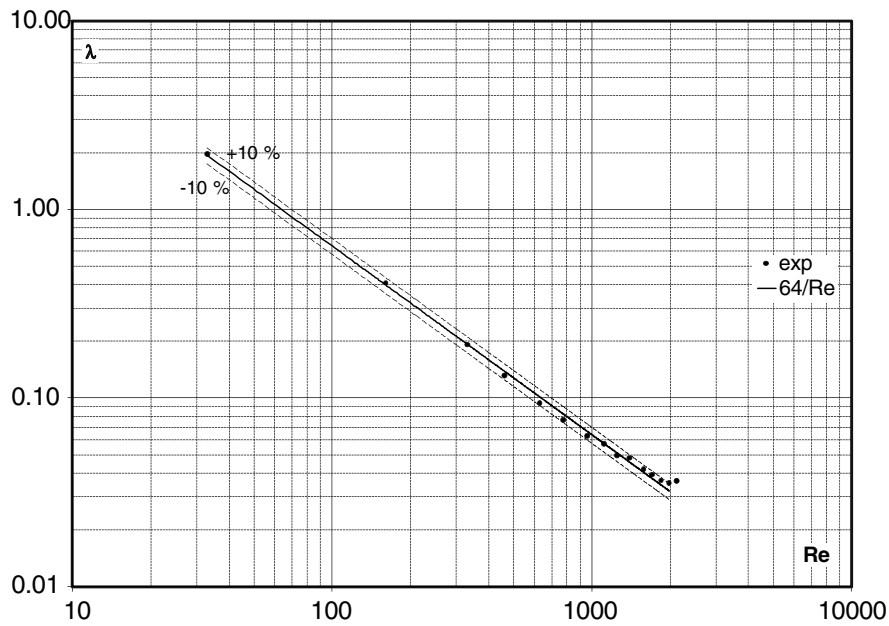


Fig. 6. Experimental frictional factor  $\lambda$  vs.  $Re$  number for water flow in minichannel with internal diameter  $d = 1.10$  mm.

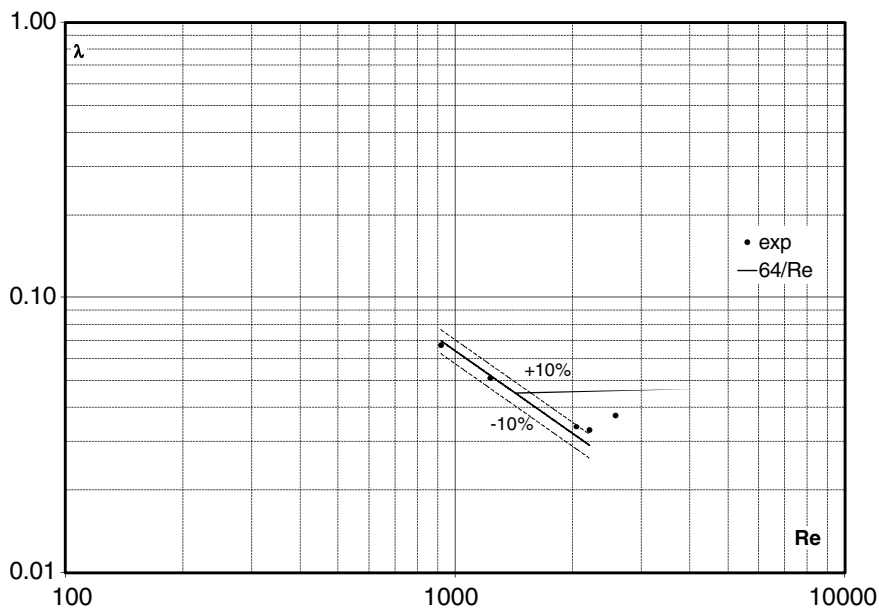


Fig. 7. Experimental frictional factor  $\lambda$  vs.  $Re$  number for air flow in a minichannel with internal diameter  $d = 0.55$  mm.

the flow-meter. Depending of the required mass flow rate, and so of the pressure on the input of the measuring section, water flew out under the influence of the pressure in the laboratory in the water system, or was aided by the used gear pump. Next, the researchers waited during the required time, i.e. until the readings of the pressure difference sensor stabilized. After this time, depending on the pipe applied and the current flow rate, in steady-state conditions, the registration of measurement was started. The measurement lasted ca. 10–20 s. In this time, data from the measuring devices were registered every 0.5 s. The average from the individual results gave quantities corresponding to a given measurement, which was further used in the calculation procedure.

**4. Pressure drop data reduction**

Reynolds number and Darcy friction factor  $\lambda$ , were determined for each case. *Reynolds* number was calculated from its definition equation with the following form:

$$Re = \frac{w \cdot d}{\nu} \tag{2}$$

where  $\nu$  is the kinematic viscosity. The measured mass flow rate  $\dot{m}$  can be written as follows:

$$\dot{m} = \rho \cdot w \cdot \frac{\pi d^2}{4}, \tag{3}$$

where  $\rho$  – water density,  $w$  – its average flow velocity.

Once the average water velocity has been determined from Eq. (3) and put in dependence Eq. (2), the following is obtained:

$$Re = \frac{4 \cdot \dot{m}}{\pi \cdot \mu \cdot d}, \tag{4}$$

where  $\mu$  is the dynamic viscosity.

From *Darcy–Weisbach* equation:

$$\frac{\Delta p}{L} = \lambda \cdot \frac{w^2 \cdot \rho}{2 \cdot d} \tag{5}$$

the Darcy friction factor  $\lambda$  is determined:

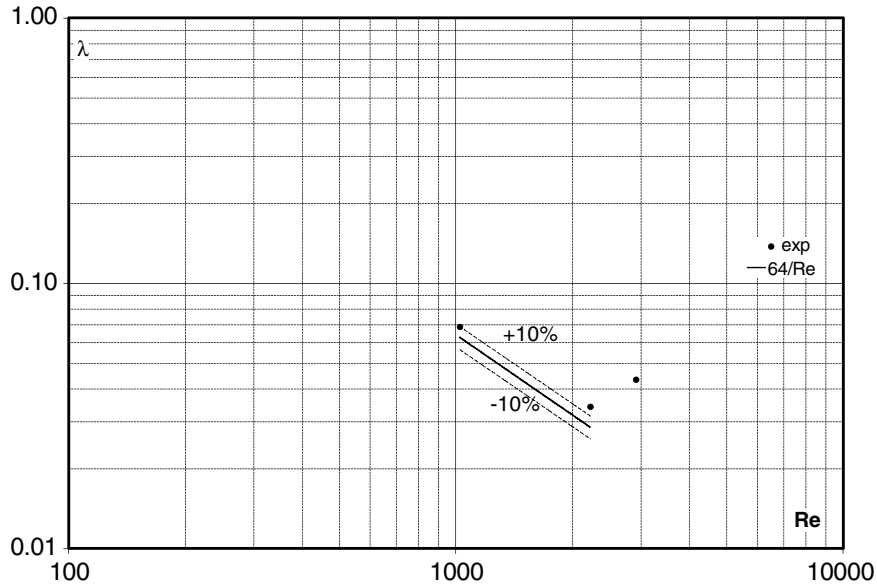


Fig. 8. Experimental frictional factor  $\lambda$  vs. *Re* number for air flow in a minichannel with internal diameter  $d = 0.64$  mm.

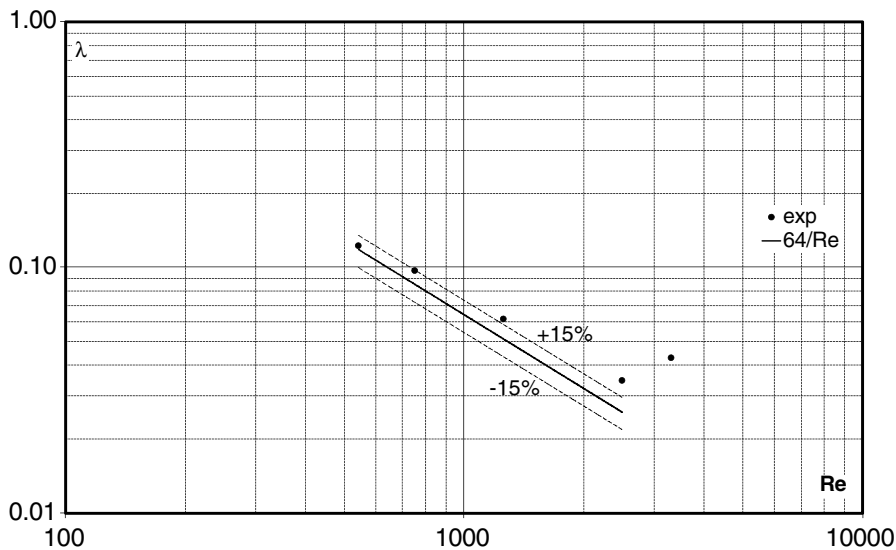


Fig. 9. Experimental frictional factor  $\lambda$  vs. *Re* number for air flow in minichannel with internal diameter  $d = 1.10$  mm.

$$\lambda_{\text{exp}} = \Delta p \frac{d}{L} \frac{2}{\rho \cdot w^2} \tag{6}$$

while the value  $(\Delta p/L)$  determines the fluid pressure drop referred to a length unit of the minichannel. Eq. (6), considering dependence Eq. (3), obtains the following form:

$$\lambda_{\text{exp}} = 0.125\pi^2 \Delta p \frac{d^5}{L} \frac{\rho}{m^2} \tag{7}$$

The theoretical value of the friction coefficient accepted in calculations for conventional channels was determined for laminar flow from the following formula:

$$\lambda_{\text{th}} = \frac{Po_{\text{th}}}{Re} = \frac{64}{Re} \tag{8}$$

where  $Po_{\text{th}} = 64$  – for channels with a circular section. In an analysis of the results of experimental investigations, *Poiseuille* number defined as [8,17]

$$Po_{\text{exp}} = \lambda_{\text{exp}} \cdot Re \tag{9}$$

was applied, as well as a normalized friction coefficient described with equation [8,17,19]:

$$C^* = \frac{(\lambda \cdot Re)_{\text{exp}}}{(\lambda \cdot Re)_{\text{th}}} \tag{10}$$

### 5. Results and discussion

Figs. 2 and 3 present results of experimental investigation of flow resistance depending of *Reynolds* number for water and air flow in three minichannels with internal diameter  $d = 0.55, 0.64$  and  $1.10$  mm. The investigations were made in the range of number  $Re = 30\text{--}6400$ . The flow resistance was expressed with the aid of value  $(\Delta p/L)$ , i.e. a water pressure drop referred to the unit of

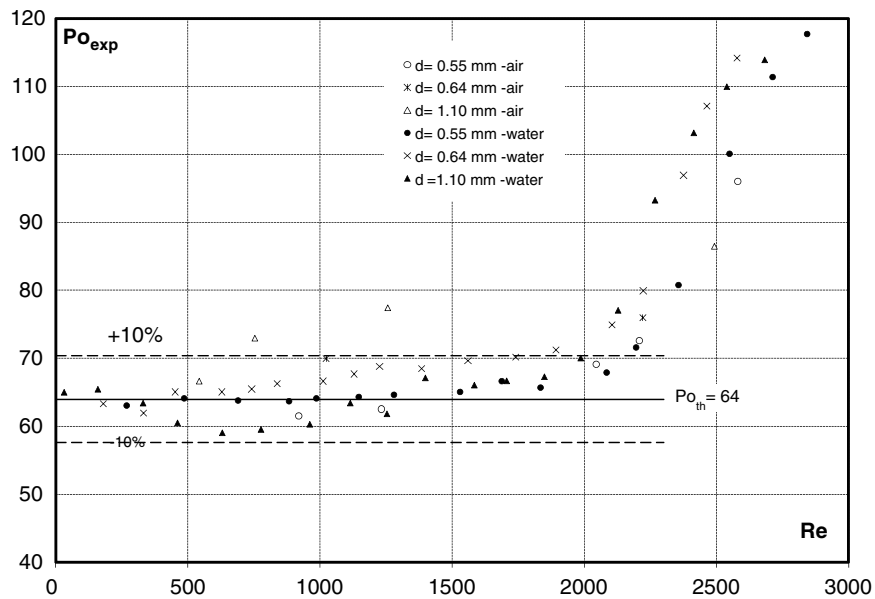


Fig. 10. Dependence of experimental *Poiseuille* number vs. *Re* number.

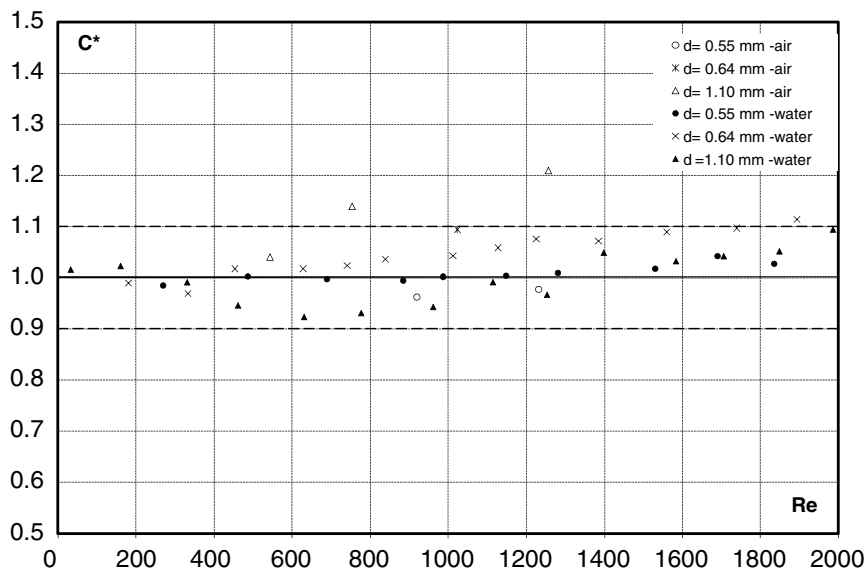


Fig. 11. Results of experimental investigations of normalized frictional factor  $C^*$  vs. *Re* number for water and air flow in minichannels in laminar flow.

the length of the measuring section [kPa/m]. An increase of  $Re$  number reflecting an increase of the water flow intensity, resulted in an increased flow resistance. For the minichannel diameter  $d = 0.55$  mm, the flow resistance value was four times greater than for a minichannel with a diameter of 1.10 mm, with the same flow intensity. It is to be noted that on the transition from laminar flow into turbulent flow, the nature of the flow of diagram  $(\Delta p/L) = f(Re)$  is subject to change. Similar trends were observed in paper [2].

Figs. 4–9 present diagrams of an experimental dependence of Darcy frictional factor  $\lambda$  vs.  $Re$  number for minichannels with internal diameters of 0.55 mm (Figs. 4 and 7), 0.64 mm (Figs. 5 and 8) and 1.10 mm (Figs. 6 and 9), respectively.

For all the cases of water and air flow in minichannels, similarly as in the case of flow in conventional channels, a linear drop of the value of frictional factor  $\lambda$  has been observed in the laminar flow. All the results of experimental investigations were in a deviation being not greater than  $\pm 10\%$  ( $\pm 20\%$  for diameter  $d = 1.10$  mm - air), as referred to the theoretical values of coefficient  $\lambda_{th}$ , described with Eq. (8). The continuous line in Figs. 4–9 is a dependence  $\lambda_{th} = f(Re)$ .

There is a clear-cut boundary of a transition from the laminar flow into turbulent flow described with  $Re$  number equal to 2000. The tendency of dependences  $\lambda_{exp} = f(Re)$  during a laminar flow of water and air in minichannels is not contradictory to its comparable tendency for conventional channels. Fig. 10 presents a dependence of experimental *Poiseuille* number (Eq. (9)) vs.  $Re$  number for each channel.

Fig. 11 presents dependence of normalized friction coefficient  $C^*$  from  $Re$  number. This coefficient was defined with Eq. (10), and thus it is described as experimental *Poiseuille* number ( $Po_{exp}$ ) divided by its theoretical value  $Po_{th} = 64$ . The experimental values of the normalized friction factor were in the range  $0.9 < C^* < 1.1$ , which should be considered as a very good compliance of the results of experimental investigations in minichannels with *Hagen – Poiseuille* law, which is valid for conventional channels.

## 6. Conclusions

The paper presents results of measurements of water flow resistance in minichannels. The measurements were made in stainless steel pipes with internal diameters of 0.55, 0.64 and 1.10 mm. The measurements were conducted in the range of Reynolds numbers  $Re = 30$  up to transition to turbulent flow. On the basis of the investigations made, one can make the following conclusions:

- a transition from laminar to turbulent flow occurs once value  $Re = 2000$  has been exceeded, which is to be observed in conventional channels, as well;
- the friction factor of laminar flow can be correctly predicted by conventional correlation  $\lambda = 64/Re$ ;

- for the tested minichannels, the possibility to apply the theory of a laminar single-phase flow in conventional channels was experimentally confirmed also for minichannels.

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## References

- [1] K. Pehlivan, I. Hassan, M. Vaillancourt, Experimental study on two-phase flow and pressure drop in millimeter-size channels, *Appl. Therm. Eng.* 26 (2006) 1506–1514.
- [2] G.P. Celata, M. Cumo, S. McPhail, G. Zummo, Characterization of fluid dynamic behaviour and channel wall effects in microtube, *Int. J. Heat Fluid Flow* 27 (2006) 135–143.
- [3] J.W. Coleman, P.E. Krause, Two phase pressure losses of R134a in microchannel tube headers with large free flow area ratios, *Exp. Therm. Fluid Sci.* 28 (2004) 123–130.
- [4] F. Kaminaga, B. Sumith, K. Matsumura, Pressure drop in capillary tube in boiling two-phase flow, in: *First International Conference on Microchannels and Minichannels*, New York, 2003, pp. 635–642.
- [5] D. Lelea, S. Nishio, K. Takano, The experimental research on microtube heat transfer and fluid flow of distilled water, *Int. J. Heat Mass Transfer* 47 (2004) 2817–2830.
- [6] M.E. Steinke, S.G. Kandlikar, Single-phase liquid friction factors in microchannels, *Int. J. Therm. Sci.* 45 (2006) 1073–1083.
- [7] Y.W. Hwang, M.S. Kim, The pressure drop in microtubes and the correlation development, *Int. J. Heat Mass Transfer* 49 (2006) 1804–1812.
- [8] G. Hetsroni, A. Mosyak, E. Pogrebnyak, L.P. Yarin, Fluid flow in micro-channels, *Int. J. Heat Mass Transfer* 48 (2005) 1982–1998.
- [9] C.-C. Wang, Y.R. Jeng, J.-J. Chien, Y.-J. Chang, Frictional performance of highly viscous fluid in minichannels, *Appl. Therm. Eng.* 24 (2004) 2243–2250.
- [10] B. Agostini, B. Watel, A. Bontemps, B. Thonon, Liquid flow friction factor and heat transfer coefficient in small channels: an experimental investigation, *Exp. Therm. Fluid Sci.* 28 (2004) 97–103.
- [11] B. Agostini, B. Watel, A. Bontemps, B. Thonon, Friction factor and heat transfer coefficient of R134a liquid flow in mini-channels, *Appl. Therm. Eng.* 22 (2002) 1821–1834.
- [12] J. Shui, R. Kulenovic, M. Groll, Heat transfer and pressure drop for flow boiling of water in narrow vertical rectangular channels, in: *First International Conference on Microchannels and Minichannels*, New York, 2003, pp. 667–673.
- [13] S.G. Kandlikar, P. Balasubramanian, Extending the applicability of the flow boiling correlations to low Reynolds number flows in microchannel, in: *First International Conference on Microchannels and Minichannels*, New York, 2003, pp. 603–608.
- [14] S.G. Kandlikar, D.A. Willistein, J. Borrelli, Experimental evaluation of pressure drop elements and fabricated nucleation sites for stabilizing flow boiling in minichannels and microchannels, in: *Third International Conference on Microchannels and Minichannels*, Toronto, Ontario, Canada, 2005, ICMM2005-75197.
- [15] S.G. Kandlikar, Microchannels and minichannels – history, terminology, classification and current research needs, in: *First International Conference on Microchannels and Minichannels*, New York, 2003, ICMM2003-100.
- [16] S.G. Kandlikar, Fundamental issues related to flow boiling in minichannels and microchannels, *Exp. Therm. Fluid Sci.* 26 (2002) 389–407.
- [17] M.E. Steinke, S.G. Kandlikar, Control and effect of dissolved air in water during flow boiling in microchannels, *Int. J. Heat Mass Transfer* 47 (2004) 1925–1935.
- [18] Z. Bonca, Czynniki chłodnicze i nosniki ciepła. Własności cieplne, chemiczne i eksploatacyjne, IPPU Masta, Gdansk, 1997.
- [19] A. Kawahara, M. Sadatami, K. Okayama, M. Kawaji, Effects of liquid properties on pressure drop of two-phase gas–liquid flow through a microchannel, in: *First International Conference on Microchannels and Minichannels*, New York, 2003, 479–486.